The table shows values of $f(x)$ and $f'(x)$ at various input values. If $g(x) = \dfrac{f(x^3)}{x}$, find $g'(2)$.

SCORE: _____ / 4 PTS

x	0	1	2	3	4	5	6	7	8	9
$f(x)$	–3	–1	0	2	1	4	6	5	2	–2
$f'(x)$	3	2	4	0	5	1	–1	–3	–5	–6

$$g'(x) = \boxed{\frac{f'(x^3)\cdot 3x^2\cdot x - f(x^3)}{x^2}} \quad 2\tfrac{1}{2}$$

THIS PRODUCT CAN BE WRITTEN DIFFERENTLY, AS LONG AS IT EQUALS –120

$$g'(2) = \frac{f'(8)\cdot 12\cdot 2 - f(8)}{4} = \boxed{\frac{(-5)\overbrace{\cancel{24}}^{12} - \cancel{2}^{1}}{\cancel{4}_2}} = \boxed{\frac{-61}{2}}$$

IF YOUR FINAL ANSWER IS CORRECT, GIVE → 1 YOURSELF THIS POINT AUTOMATICALLY

$\frac{1}{2}$

Find $\frac{dy}{dx}$ if $\sin\frac{x}{y} = x^3y^2$.

SCORE: _____ / 6 PTS

$$\left(\cos\frac{x}{y}\right)\left(\frac{y-xy'}{y^2}\right) = 3x^2y^2 + 2x^3yy'$$

1 1 1 1

$$\left(\cos\frac{x}{y}\right)(y-xy') = 3x^2y^4 + 2x^3y^3y'$$

$$y\cos\frac{x}{y} - \left(x\cos\frac{x}{y}\right)y' = 3x^2y^4 + 2x^3y^3y'$$

$$y\cos\frac{x}{y} - 3x^2y^4 = 2x^3y^3y' + \left(x\cos\frac{x}{y}\right)y'$$

$$= \left(2x^3y^3 + x\cos\frac{x}{y}\right)y'$$

1

$$\frac{dy}{dx} = \boxed{\frac{y\cos\frac{x}{y} - 3x^2y^4}{2x^3y^3 + x\cos\frac{x}{y}}}$$

1

TALK TO ME IF YOU REWROTE THE EQUATION FIRST

Prove the derivative of $\cos^{-1} x$. **Show all steps and justifications.** SCORE: _____ / 4 PTS

$y = \cos^{-1} x \longrightarrow y \in [0, \pi] \longrightarrow \sin y \geq 0$ 1

$\cos y = x$ 1

so $\sin y = \sqrt{1 - \cos^2 y}$ $\frac{1}{2}$

$= \sqrt{1 - x^2}$

$-\sin y \frac{dy}{dx} = 1$ $\frac{1}{2}$

$\frac{dy}{dx} = -\frac{1}{\sin y} = -\frac{1}{\sqrt{1-x^2}}$ $\frac{1}{2}$ $\frac{1}{2}$

If the position function of an object is given by $f(t) = \tan^{-1} t^3$, find the acceleration function. **SCORE: _____ / 5 PTS**

$$f'(t) = \frac{1}{1+(t^3)^2} 3t^2 = \boxed{\frac{3t^2}{1+t^6}} \quad 2$$

$$f''(t) = \boxed{\frac{6t(1+t^6) - 3t^2(6t^5)}{(1+t^6)^2}} \quad 2$$

$$= \frac{6t + 6t^7 - 18t^7}{(1+t^6)^2}$$

$$= \boxed{\frac{6t - 12t^7}{(1+t^6)^2}} = \frac{6t(1-2t^6)}{(1+t^6)^2} \quad 1$$

If $f(x) = (\csc x)^{x^4}$, find $f'(x)$.

SCORE: _____ / 6 PTS

$y = (\csc x)^{x^4}$

$\boxed{\ln y = x^4 \ln \csc x}$ 1½

$\boxed{\frac{1}{y}\frac{dy}{dx}} = \boxed{4x^3 \ln \csc x + x^4 \frac{1}{\cancel{\csc x}}(-\cancel{\csc x} \cot x)}$ 2½

1

$\frac{dy}{dx} = \boxed{(\csc x)^{x^4}(4x^3 \ln \csc x - x^4 \cot x)}$ 1

$= x^3 (\csc x)^{x^4}(4 \ln \csc x - x \cot x)$

If $f(x) = \ln(\sqrt{x^2+1} - x)$, find $f'(x)$. **<u>NOTE: In the final simplified answer, x only appears once.</u>** SCORE: _____ / 5 PTS

$$f'(x) = \frac{1}{\sqrt{x^2+1}-x} \cdot \left(\frac{1}{\cancel{2}\sqrt{x^2+1}} \cdot \cancel{2}x - 1 \right)$$

$$= \text{(1)}\ \boxed{\frac{1}{\sqrt{x^2+1}-x}} \boxed{\left(\frac{x}{\sqrt{x^2+1}} - 1 \right)}\ \text{(2)}$$

$$= \frac{1}{\sqrt{x^2+1}-x} \boxed{\frac{x - \sqrt{x^2+1}}{\sqrt{x^2+1}}}\ \text{(1)}$$

$$= \boxed{\frac{-1}{\sqrt{x^2+1}}}\ \text{(1)}$$